

***BENCHMARK FLOW PROBLEMS -
A ROUND-ROBIN EXERCISE***

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**Presented at:
DIERS Users Group Meeting
San Antonio, Texas**

October 20-22, 2008

Critical Discharge Round-Robin Exercise

For benchmarking purpose.

Thermodynamic equilibrium assumption.

Homogeneous flow (no slip).

Three inlet qualities (0.0001, 0.01, 0.1).

Case I (Cyclohexane) at 10 bara.

Case II (20% mole Ethane in n-Heptane) at 10 bara.

Case III (2.5 % mole N₂ in Cyclohexane) at 33 bara.

Proposed choked flow problems

Case	Liquid Composition (mole)	P_o (bar)	T_o (°C)	x_o (vapor mass)
Ia	100% c-C6	10	182.3	0.0001 (bubble pt)
Ib	100% c-C6	10	182.3	0.01
Ic	100% c-C6	10	182.3	0.1
IIa	20% C2/n-C7	10	51.9	0.0001 (bubble pt)
IIb	20% C2/n-C7	10	51.9	0.01
IIc	20% C2/n-C7	10	51.9	0.1
IIIa	2.5% N ₂ /c-C6	33	25	0.0001 (bubble pt)
IIIb	2.5% N ₂ /c-C6	33	25	0.01
IIIc	2.5% N ₂ /c-C6	33	25	0.1

Data Submission

Invite EDUG.

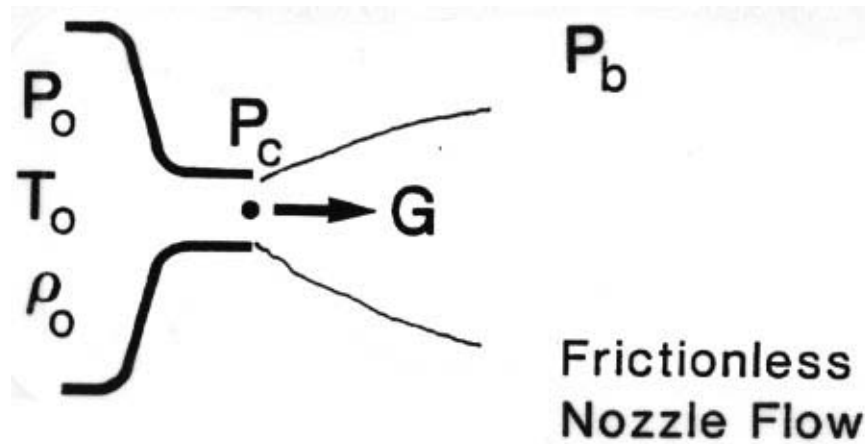
Two weeks before next US DIERS Users Group Mtg.

E-mail all relevant results (outputs) together with a summary sheet listing the mass flux G (kg/m²s) and choking pressure (bar) or ratio.

Post invitation and problem specification in DIERS website.

Two-Phase Flow in Nozzle

DIERS UG San Antonio Mtg./October 20-22 2008



From Thermodynamics:

First Law (Energy Equation)

$$h_o = h + \frac{u^2}{2} = h + \frac{(Gv)^2}{2} \quad (1)$$

Second Law (Adiabatic Reversible)

$$dh = T dS + v dP \quad (2)$$

Combining:

$$G = \frac{\sqrt{2(h_o - h)}}{v} = \frac{\left[2 \int_{P_o}^P -v dP \right]^{0.5}}{v} \quad (3)$$

Two-Phase Simple Fluid Expansion

$$\int \frac{dP}{\rho} \quad \text{or} \quad \int v \, dP$$

$$\frac{v}{v_o} = \omega \left(\frac{P_o}{P} - 1 \right) + 1 \quad , \quad \frac{\rho_o}{\rho} = \omega \left(\frac{P_o}{P} - 1 \right) + 1$$

Flashing flow (phase change):

$$\omega = \alpha_o \left(1 - 2 \frac{P_o v_{fgo}}{h_{fgo}} \right) + \frac{C_{pf} T_o P_o}{v_o} \left(\frac{v_{fgo}}{h_{fgo}} \right)^2$$

Non – flashing flow (no phase change):

$$\omega = \frac{\alpha_o}{k}$$

Alternate Method for Multicomponent System with wide boiling pt. range (Nazario-Leung 1992 paper)

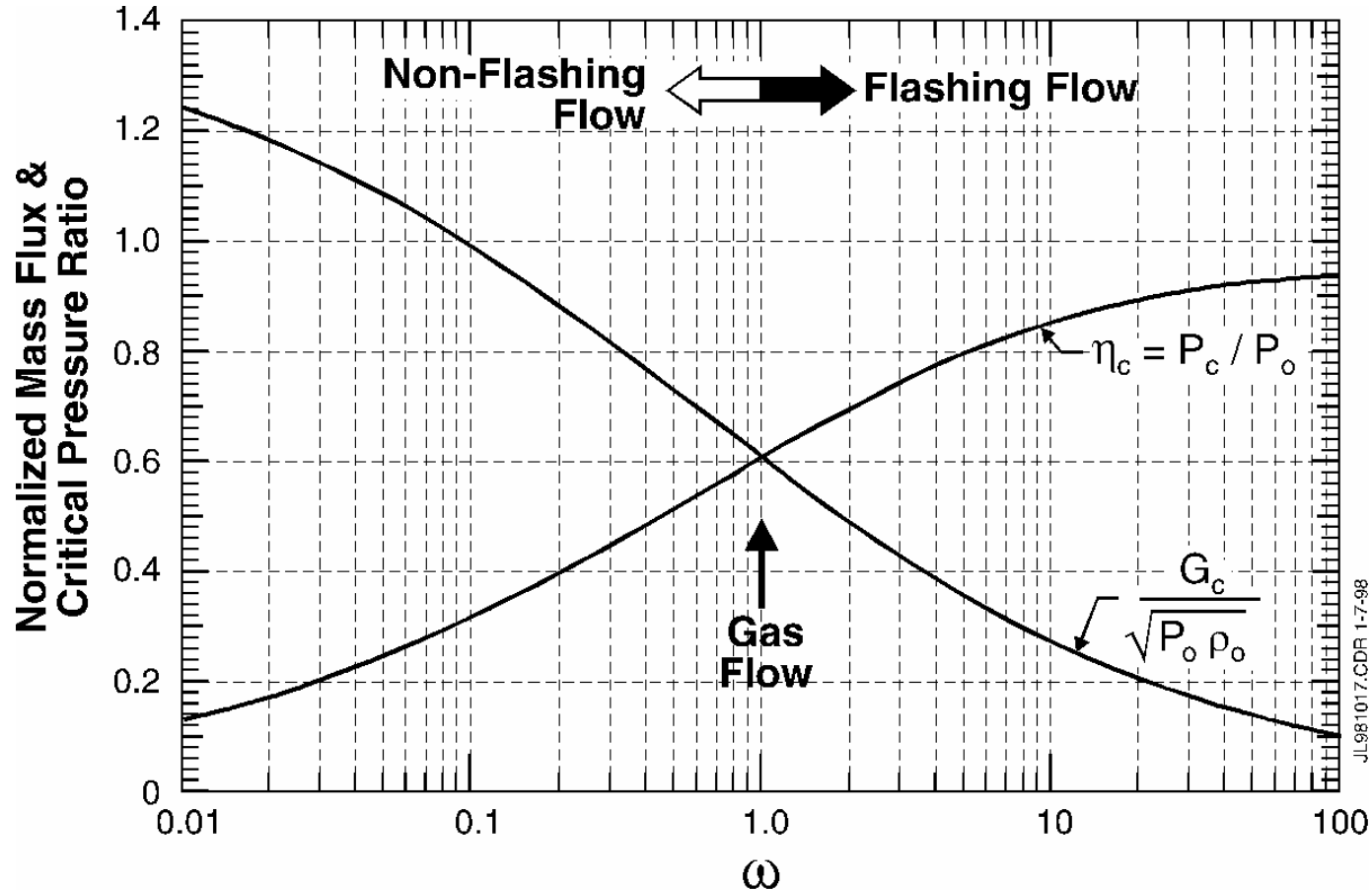
- Use the original EOS as suggested by

$$\frac{v}{v_0} = \omega \left(\frac{P_0}{P} - 1 \right) + 1 \quad (36)$$

- Re-defining ω to account for realistic flashing behavior by

$$\omega = \frac{\left(\frac{v}{v_0} - 1 \right)}{\left(\frac{P_0}{P} - 1 \right)} \quad (37)$$

- Do a multicomponent flash calculation at 90% of the inlet pressure P_0 . Calculate v and ω .
- Use generalized $G^* - \omega$ correlation to obtain the flashing flow rate per unit area.



Flashing and non-flashing choked flow through nozzles - a unified chart. Merging the two analytical solutions, Eqs. (10) and (12). Source: Leung, 1990a and 1990b.

L. L. Simpson empirical fit for P_c/P_o given by [Process Safety Progress 22(1),27(2003)]

$$P_c / P_o = \left[1 + (1.0446 - 0.0093431 \omega^{0.5}) \omega^{-0.56261} \right]^{(-0.70356 + 0.014685 \ln \omega)} \quad -8-$$

<u>Participants</u>	<u>Flash Routine</u>	<u>Method of Calculation</u>
Joseph Leung (Leung Inc.)	in-house	ΔH , vdP, omega
Greg Hendrickson (Chevron Phillips)	Aspen Plus	ΔH
James Goom (Aspen Tech.)	Aspen Dynamics	ΔH , vdP (Simpson)
Nikita Podlevskikh (CISP)	VENT (TSS)	$G = (dv/dP)_s^{-1/2}$
Warren Greenfield (ISP)	SimSci PRO/II	ΔH
Bob D'Alessandro (Evonik-Degussa)	Aspen Plus	vdP
Bob D'Alessandro (Evonik-Degussa)	SuperChem	vdP
Harold Fisher (Fisher Inc.)	Proprietary	vdP (Simpson)
Georges Melhem (Iomosaic)	SuperChem	vdP
Enio Kumpinsky (Ashland)	SuperChem	vdP
Dan Smith (Albermarle)	in-house	vdP (Simpson)
	in-house	omega 0.9
	Chemcad	unknown

100% c-C6, 10 bar, 182.3°C

Method (initials)	Case Ia $x_o = 0.0001$		Case Ib $x_o = 0.01$		Case Ic $x_o = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
ω_o (JL)	6140	0.863	6020	0.849	5190	0.778
$\omega_{0.9}$ (JL)	6160	0.863	6020	0.849	5210	0.777
vdP (JL)	6120	0.870	5990	0.857	5180	0.780
Aspen ΔH (GH)	5940	0.845	5810	0.835	5100	0.794
Aspen ΔH (JG)	6050	0.863	5930	0.850	5160	0.778
Aspen vdP (JG)	6090	0.864	5960	0.852	5170	0.780
Vent (NP)	6150	0.867	6010	0.853	5200	0.795
Simsci ΔH (WG)	6050	0.860	NA	NA	NA	NA
Aspen vdP (RD)	6130	0.870	5980	0.860	5120	0.790
SuperChem vdP (RD)	6060	0.875	5920	0.866	5100	0.784
SuperChem vdP (GH)	6080	0.875	5980	0.866	5160	0.784
vdP (HF)	5850	0.873	5720	0.862	4940	0.793
Mean Value	6070	0.866	5940	0.854	5140	0.785
Std. Dev.	90	0.008	70	0.009	80	0.007

20% C2/n-C7 (Liq. Comp.), 10 bar, 51.9°C

Method (initials)	Case IIa $x_o = 0.0001$		Case IIb $x_o = 0.01$		Case IIc $x_o = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
$\omega_{0.9}$ (JL)	9850	0.762	8940	0.733	5650	0.652
vdP (JL)	9780	0.768	8890	0.738	5650	0.650
Aspen ΔH (GH)	9700	0.783	8800	0.731	5600	0.659
Aspen ΔH (JG)	9800	0.762	8910	0.733	5650	0.651
Aspen vdP (JG)	9810	0.761	8910	0.732	5650	0.648
Vent (NP)	9180	0.750	8500	0.717	5670	0.640
Simsci ΔH (WG)	9870	0.755	NA	NA	NA	NA
Aspen vdP (RD)	9920	0.760	8970	0.730	5680	0.650
SuperChem vdP (RD)	9910	0.757	8950	0.731	5650	0.653
SuperChem vdP (GH)	9970	0.750	9040	0.725	5700	0.651
vdP (HG)	9840	0.756	8880	0.730	5510	0.656
Mean Value	9780	0.760	8880	0.730	5640	0.651
Std. Dev.	210	0.009	150	0.006	50	0.005

2.5% N₂/c-C6 (Liq. Comp.), 33 bar, 25°C

Method (initials)	Case IIIa $x_o = 0.0001$		Case IIIb $x_o = 0.01$		Case IIIc $x_o = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
$\omega_{0.9}$ (JL)	47190	0.377	38050	0.456	19630	0.567
vdP (JL)	46600	0.390	37800	0.460	19630	0.566
Aspen ΔH (GH)	45000	0.382	37000	0.458	19550	0.565
Aspen ΔH (JG)	43880	0.377	36090	0.453	20060	0.566
Aspen vdP (JG)	43620	0.377	35970	0.449	20190	0.560
Vent (NP)	44800	0.355	40730	0.403	22040	0.494
Simsci ΔH (WG)	44990	0.385	NA	NA	NA	NA
Aspen vdP (RD)	45900	0.382	37390	0.455	19410	0.567
SuperChem (GM)	45850	0.378	37430	0.464	19330	0.567
SuperChem (GH)	46570	0.387	37440	0.462	19650	0.568
vdP (HF)	45880	0.382	37470	0.458	19390	0.570
Mean	45480	0.379	37540	0.452	19890	0.559
Std. Dev.	1130	0.009	1310	0.018	810	0.023

20% C2/n-C7 (Liq. Comp.), 10 bar, 51.9°C

Sensitivity to VLE model, ΔH , vdP (Aspen)

Method (initials)	Case IIIa $x_0 = 0.0001$		Case IIIb $x_0 = 0.01$		Case IIIc $x_0 = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
Aspen ΔH , kij = 0.0067 (JG)	9800	0.762	8910	0.733	5650	0.651
Aspen vdP, kij = 0.0067 (JG)	9810	0.761	8910	0.732	5650	0.648
Mean Value	9780	0.760	8880	0.730	5640	0.651
Std. Dev.	210	0.009	150	0.006	50	0.005

2.5% N₂/c-C6 (Liq. Comp.), 33 bar, 25°C

Sensitivity to VLE model (Aspen)

Method (initials)	Case IIIa $x_0 = 0.0001$		Case IIIb $x_0 = 0.01$		Case IIIc $x_0 = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
Aspen ΔH , NRTL (JG)	43880	0.377	36090	0.453	20060	0.566
Aspen vdP, NRTL (JG)	43620	0.377	35970	0.449	20190	0.560
Aspen ΔH , kij=0.176 (JG)	43970	0.375	36080	0.452	19950	0.566
Aspen ΔH , kij= (GH)	45000	0.382	37000	0.458	19550	0.565
Mean (ALL submitted)	45480	0.379	37540	0.452	19890	0.559
Std. Dev. (All submitted)	1130	0.009	1310	0.018	810	0.023

Sensitivity to liquid density for bubble pt mass flux evaluation

Method (initials)	Case Ia $x_o = 0.0001$ Cyclo-C6		Case IIa $x_o = 0.0001$ C2-nC7		Case IIIa $x_o = 0.0001$ N2-cycloC6	
	Liquid density (kg/m ³)	G_c (kg/m ² s)	Liquid density (kg/m ³)	G_c (kg/m ² s)	Liquid density (kg/m ³)	G_c (kg/m ² s)
$\omega_{0.9}$ (JL) PR EOS liquid density	619	6160	627	9780	812	46600
Aspen ΔH (JG) API liquid density (default)	600	5940	622	9700	767	45000
Aspen vdP (JG) API liquid density (default)	600	6090	624	9810	769	43970

Case I - Cyclohexane

One-component

$P_o = 10$ bara (145 psia), T_o (saturation)

DIPPR (P-T, liquid density, latent heat, etc.)

advantage: accurate P_{sat} , ρ_L , ΔH_{VL}

disadvantage: no data on H_L , H_V , S_L , S_V

EOS (SRK, PR) -

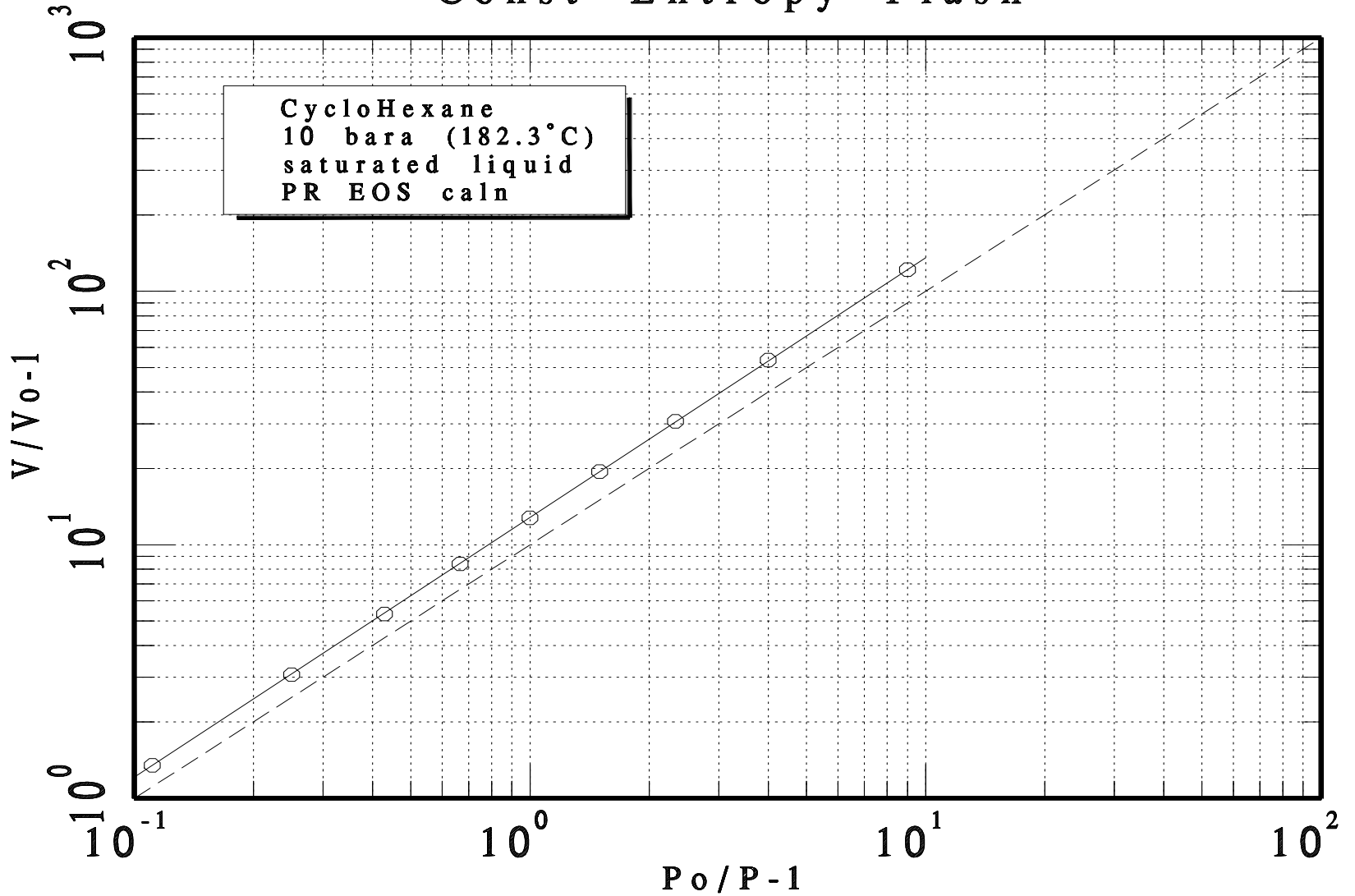
advantage: Z_L , Z_V , H_L , H_V , S_L , S_V

disadvantage: Z_L (5% off)

Case 1 – cyclo-C6 constant S flash

P BAR	T C	VL m3/kmol	VV m3/kmol	V/F mole quality	V 2ph m3/kmol
10	182.28	0.136	3.069	0.0	0.136
9	176.25	0.1337	3.424	0.0564	0.3193
8	169.69	0.1313	3.863	0.1131	0.5533
7	162.49	0.1289	4.422	0.1704	0.8603
6	154.46	0.1264	5.157	0.2289	1.2779
5	145.36	0.1239	6.171	0.2892	1.8727
4	134.76	0.1212	7.665	0.3524	2.7794
3	121.91	0.1183	10.102	0.4201	4.3121
2	105.21	0.1149	14.832	0.4955	7.4066
1	79.96	0.1107	28.372	0.5865	16.6852

Const Entropy Flash



Case I – Cyclo-C6 Preliminary Solution (Long Form)

PR EOS ($P_c = 40.8$ bar, $T_c = 553.8$ K, acentric factor = 0.208)

10 bar, 182.3°C, $x_o = 0.0$ (0.0001)

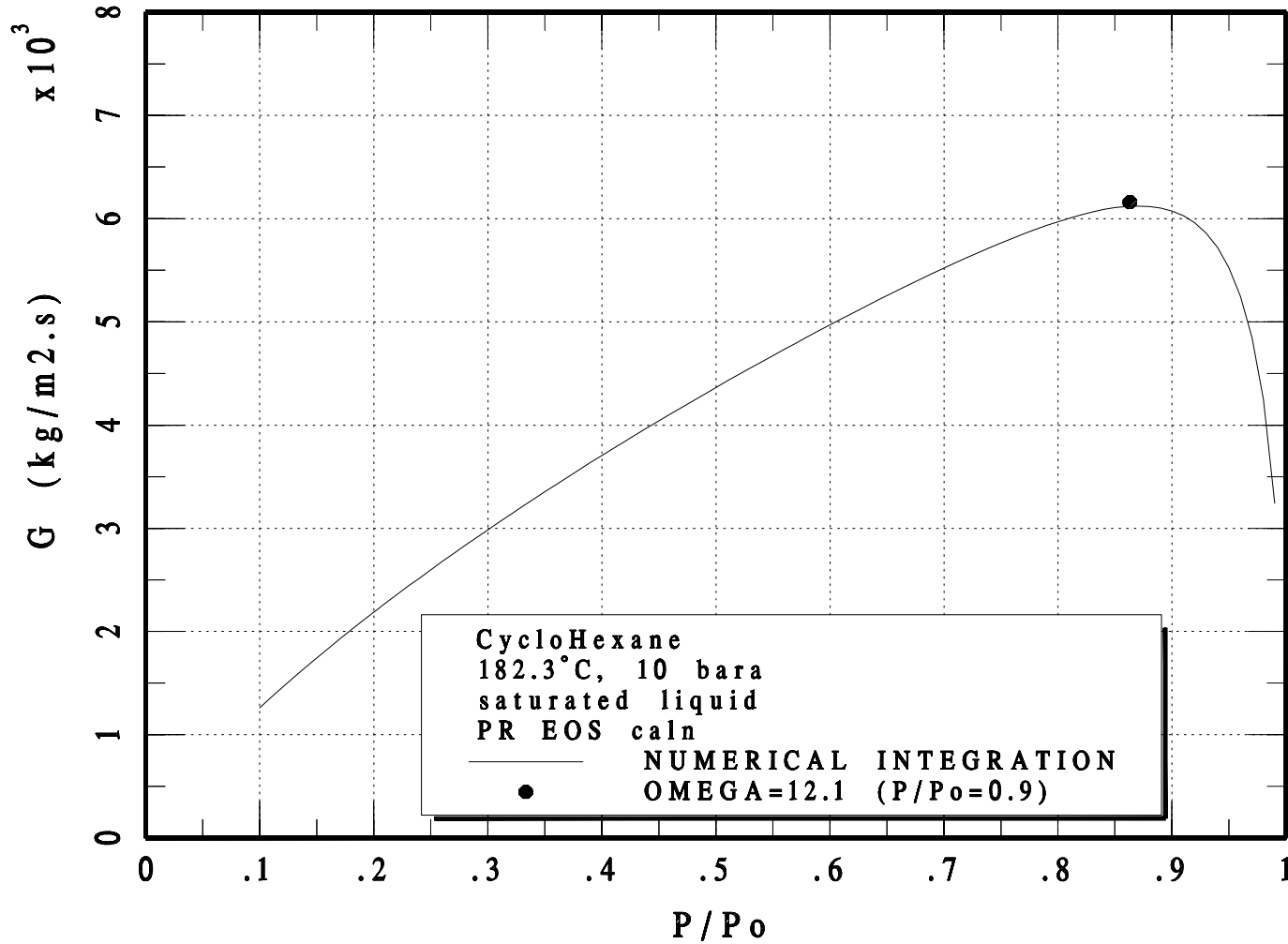
Constant S flash to 9 bar, 8 bar and so on

Curve fit P-v data

$$\left(\frac{v}{v_o} - 1 \right) = 12.8 \left(\frac{P_o}{P} - 1 \right)^{1.025}$$

Numerical integration of $\left[2 \int_{P_o}^{P_t} -v \, dP \right]^{1/2} / v_t$ to seek max G

FLOW THRU NOZZLE (Const S)



Case II – 20% mole Ethane / n-Heptane

**Bubble-point prediction to match available literature
VLE data on C2-C7 binary system -**

Kays, I&EC 30, 459 (1938)

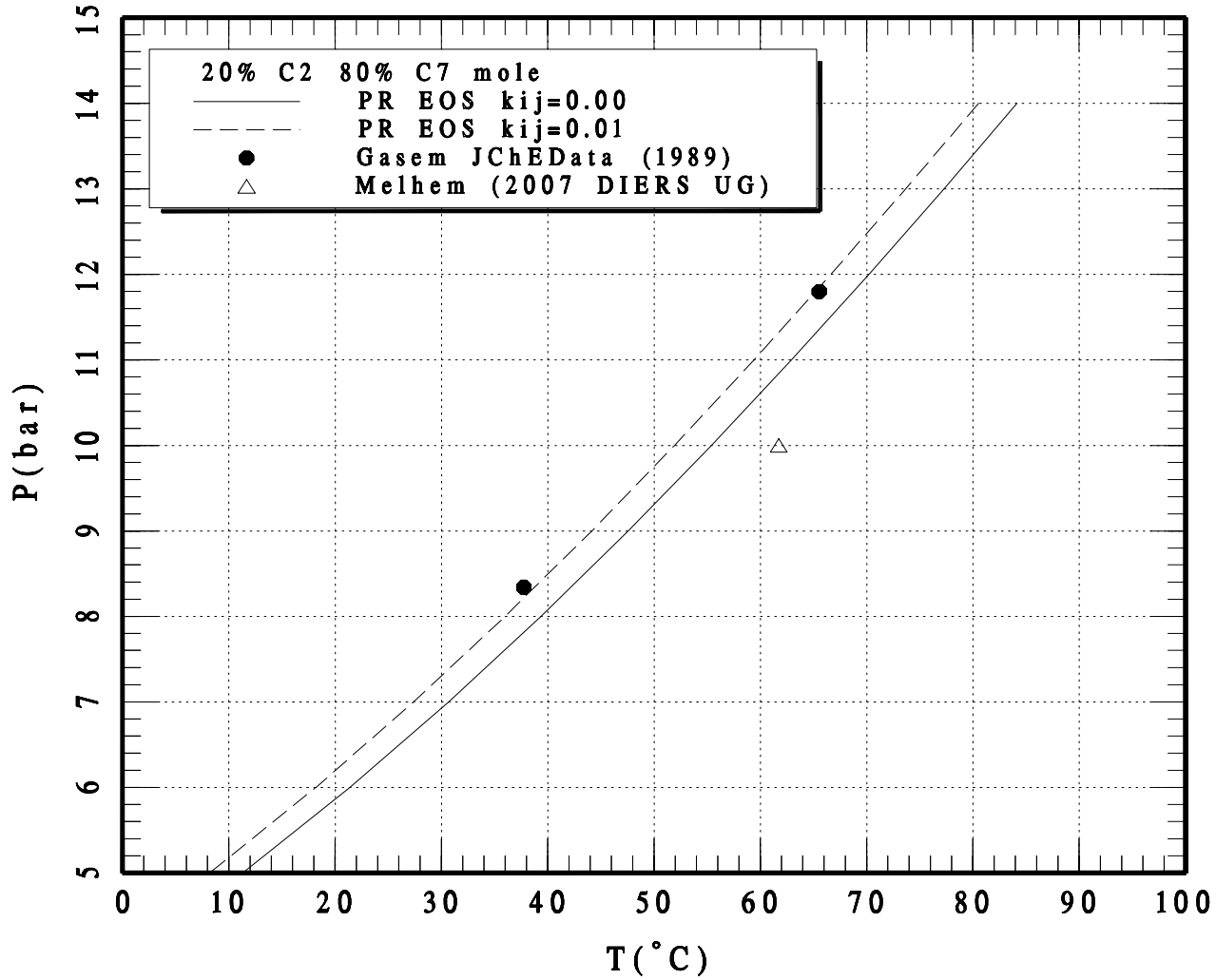
Gasem et. al., J Ch Eng Data, 34, 397 (1989)

PR EOS for C2-C7 binary

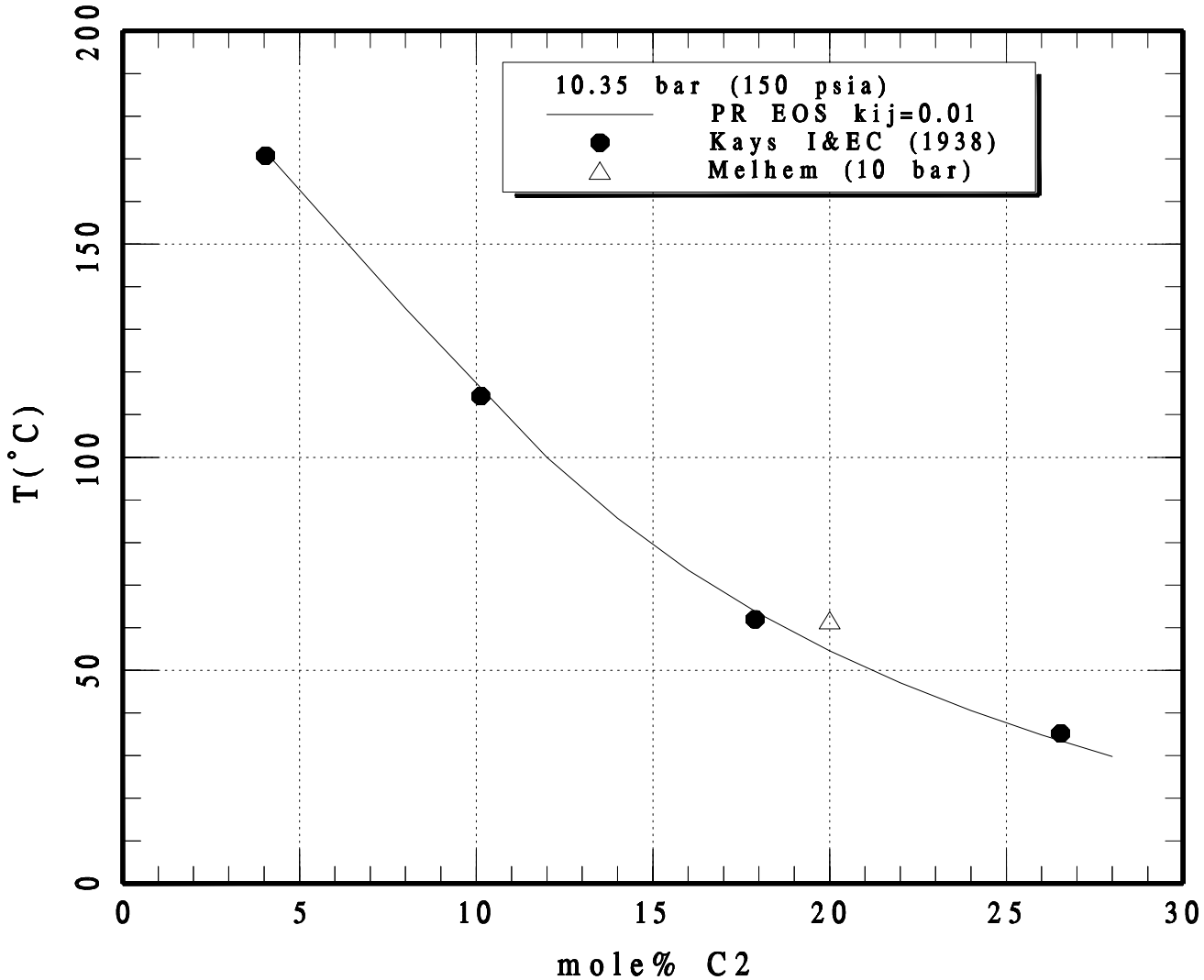
$k_{12} = 0.01$ (best fit)

(caution - different value for SRK)

PR-EOS BUBBLE PT PREDICTION



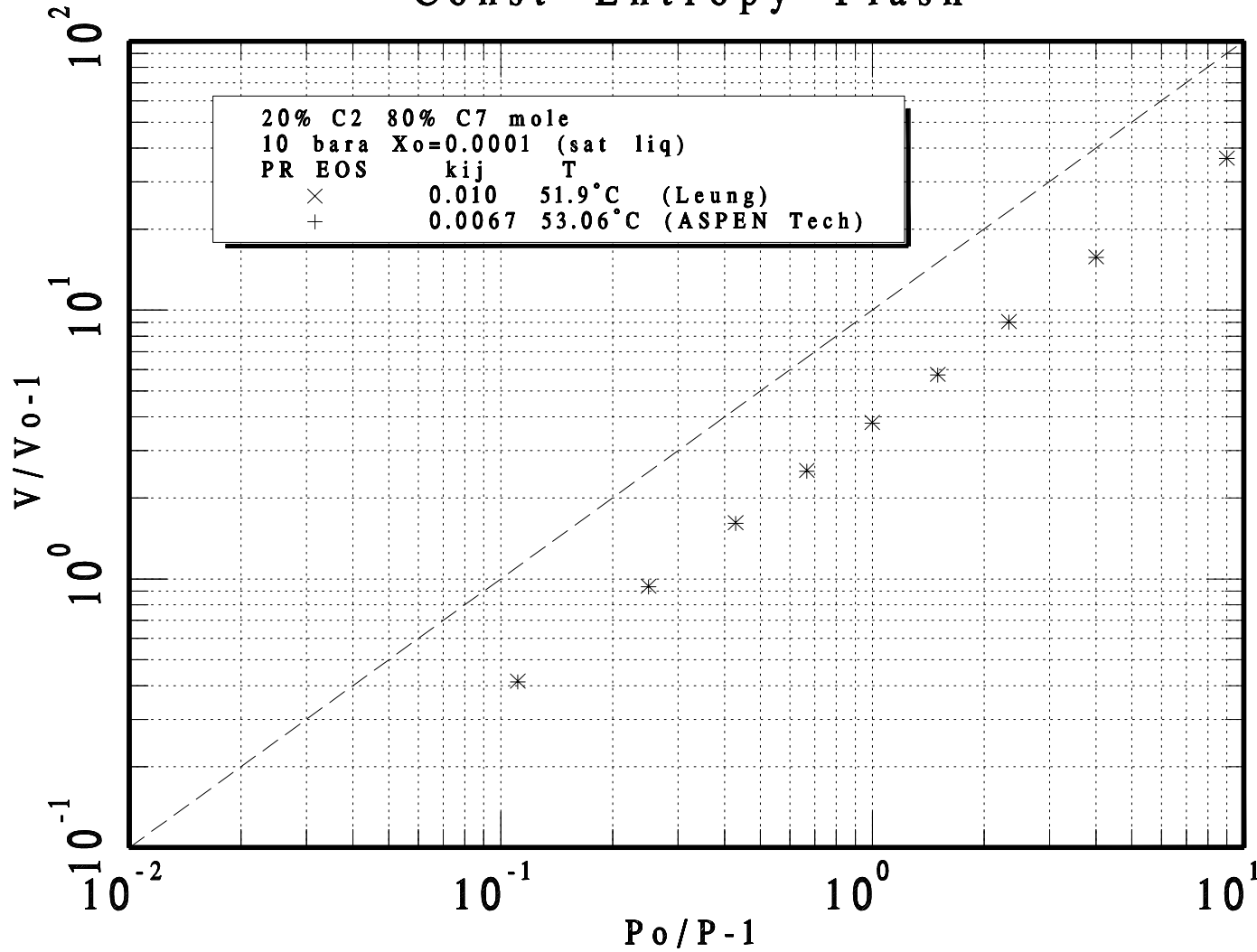
PR-EOS BUBBLE PT PREDICTION



Case IIa – C2 / n-C7 constant S flash

P	T	X (C2)	Y (C2)	VL	VV	L/F mole
BAR	C	mole fr	mole fr	m ³ /kmol	m ³ /kmol	liq/feed
10	51.92	0.2000	0.9752	0.13725	2.5025	0.99988
9	50.78	0.18317	0.97420	0.13837	2.7906	0.97872
8	49.58	0.16586	0.97284	0.13953	3.1245	0.96415
7	48.31	0.14794	0.97106	0.14074	3.6120	0.93676
6	46.94	0.12940	0.96865	0.14200	4.2264	0.91588
5	45.45	0.11012	0.96528	0.14330	5.0847	0.89490
4	43.75	0.09010	0.96031	0.14464	6.3683	0.87371
3	41.72	0.06922	0.95233	0.14600	8.4991	0.85191
2	39.02	0.04739	0.93754	0.14735	12.734	0.82856
1	34.35	0.02443	0.90041	0.14848	25.283	0.79958

Const Entropy Flash



Case II C2-C7 Preliminary Solution (Long Form)

PR EOS (20% mole C2 in n-C7)

10 bara, 51.9°C, $x_o = 0.0$ (0.0001)

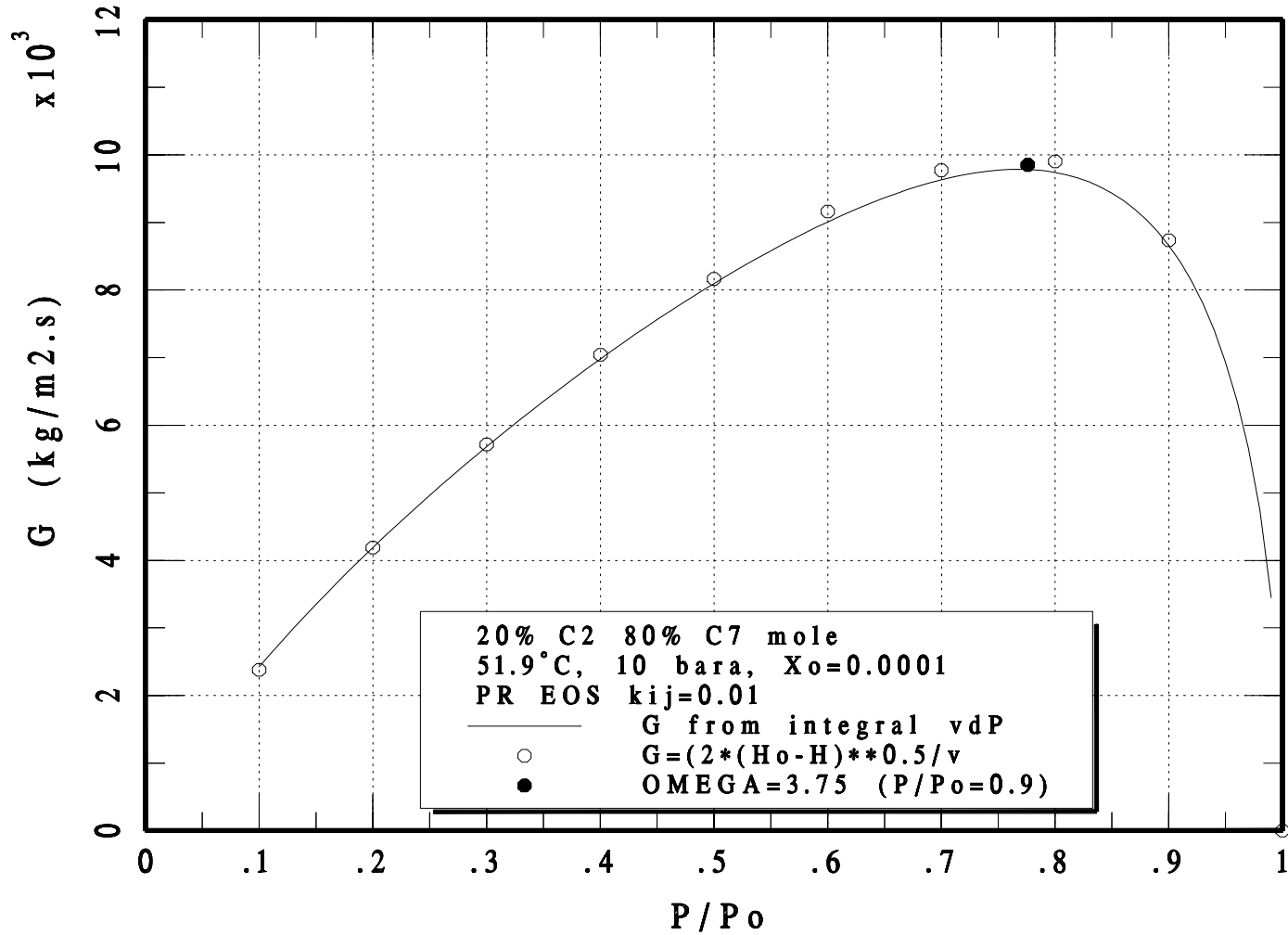
Constant S flash to lower pressures

Curve fit P-v data

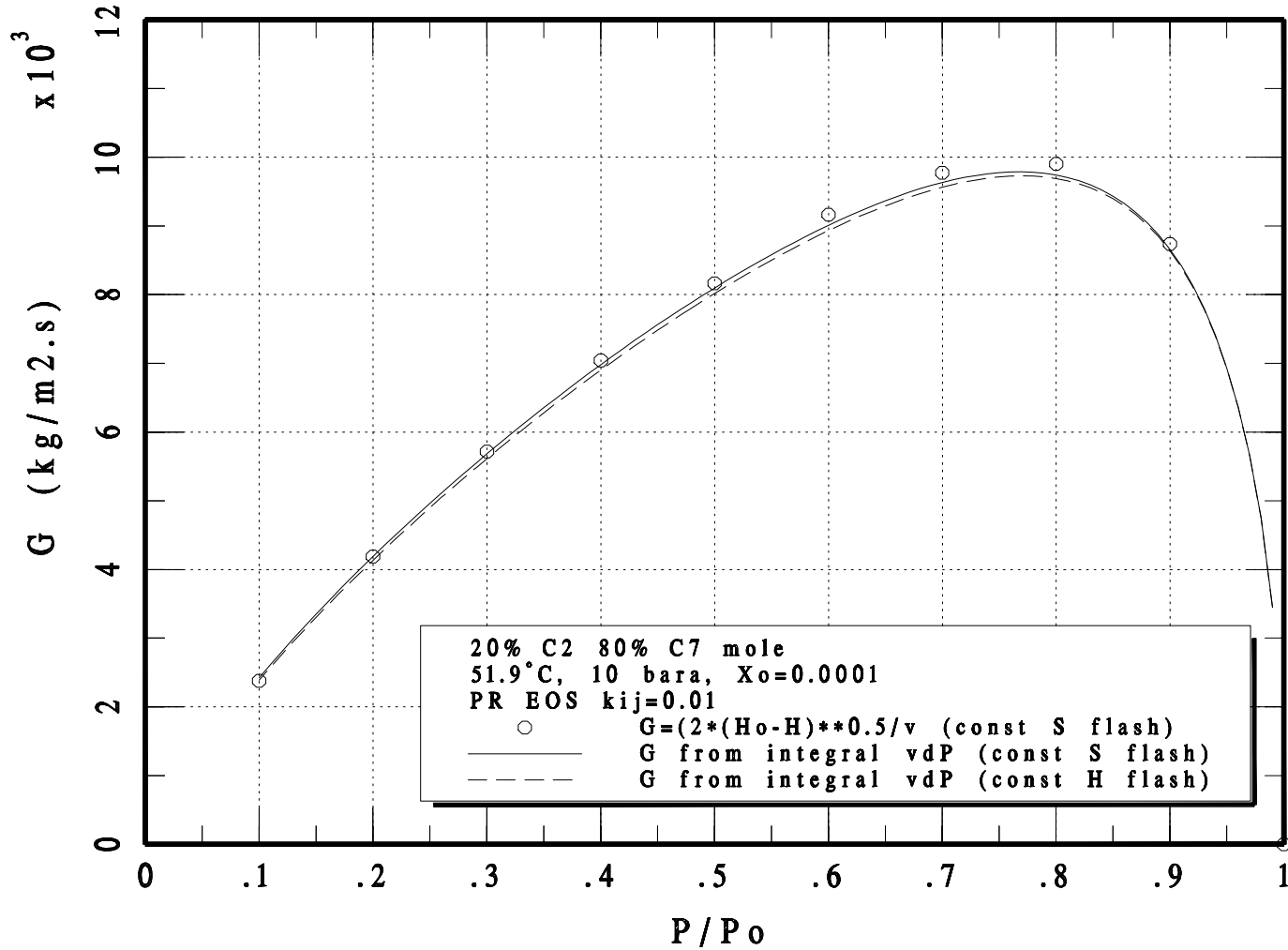
$$\left(\frac{v}{v_o} - 1 \right) = 3.87 \left(\frac{P_o}{P} - 1 \right)^{1.015}$$

Numerical integration of $\left[2 \int_{P_o}^{P_t} -v \, dP \right]^{1/2} / v_t$ to seek
max G

FLOW THRU NOZZLE (Const S)



FLOW THRU NOZZLE (Const S)



Case III - N₂ / Cyclo-C6

Bubble-point prediction using PR EOS to match available
N₂ solubility data in cyclo-C6 :

Hildebrand, JACS, 71, 3147 (1949)

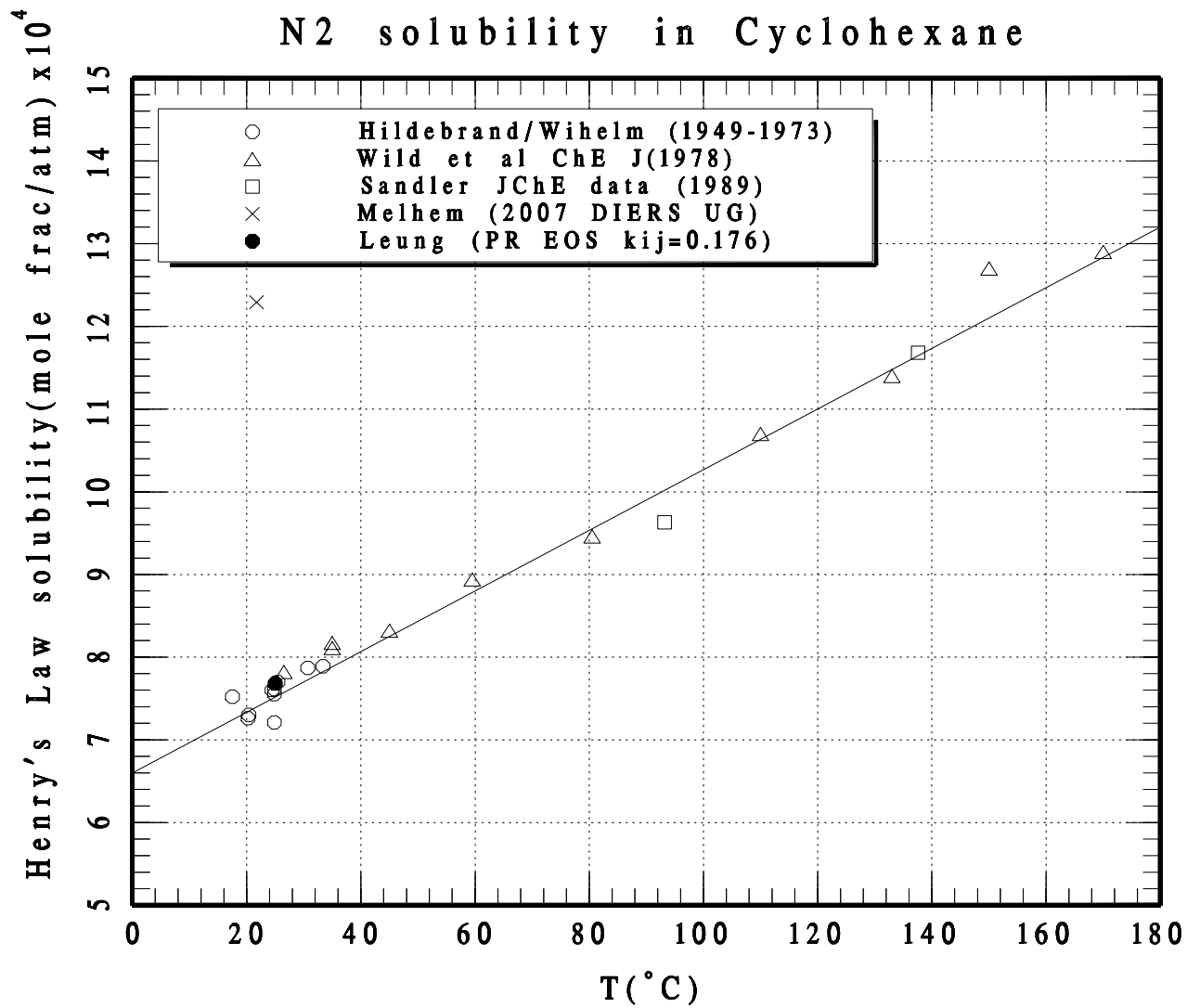
Hildebrand, I&EC Fund 6, 130 (1967)

Wilhelm, Chem Rev, 1, 73 (1973)

Wild, Ch E J, 15, 209 (1978)

Sandler, J Ch E Data, 34, 419 (1989)

Determine initial condition and k_{12} (BIP) for this mixture.



PR EOS for N₂ / cyclo-C6 binary

$$P_o = 33 \text{ bar}$$

$$T_o = 25^\circ\text{C}$$

Henry's Law constant for N₂ / cyclo-C6 pair -

$$7.68 \times 10^{-4} \text{ mole frac/atm (solubility form: } x = H P)$$

$$1320 \text{ bar/mole frac (volatility form: } P = H^* x)$$

$$\begin{aligned} \text{N}_2 \text{ mole frac} &= 7.68 \times 10^{-4} (33 \times 0.9864 \text{ atm}) \\ &= 0.025, 2.5\% \end{aligned}$$

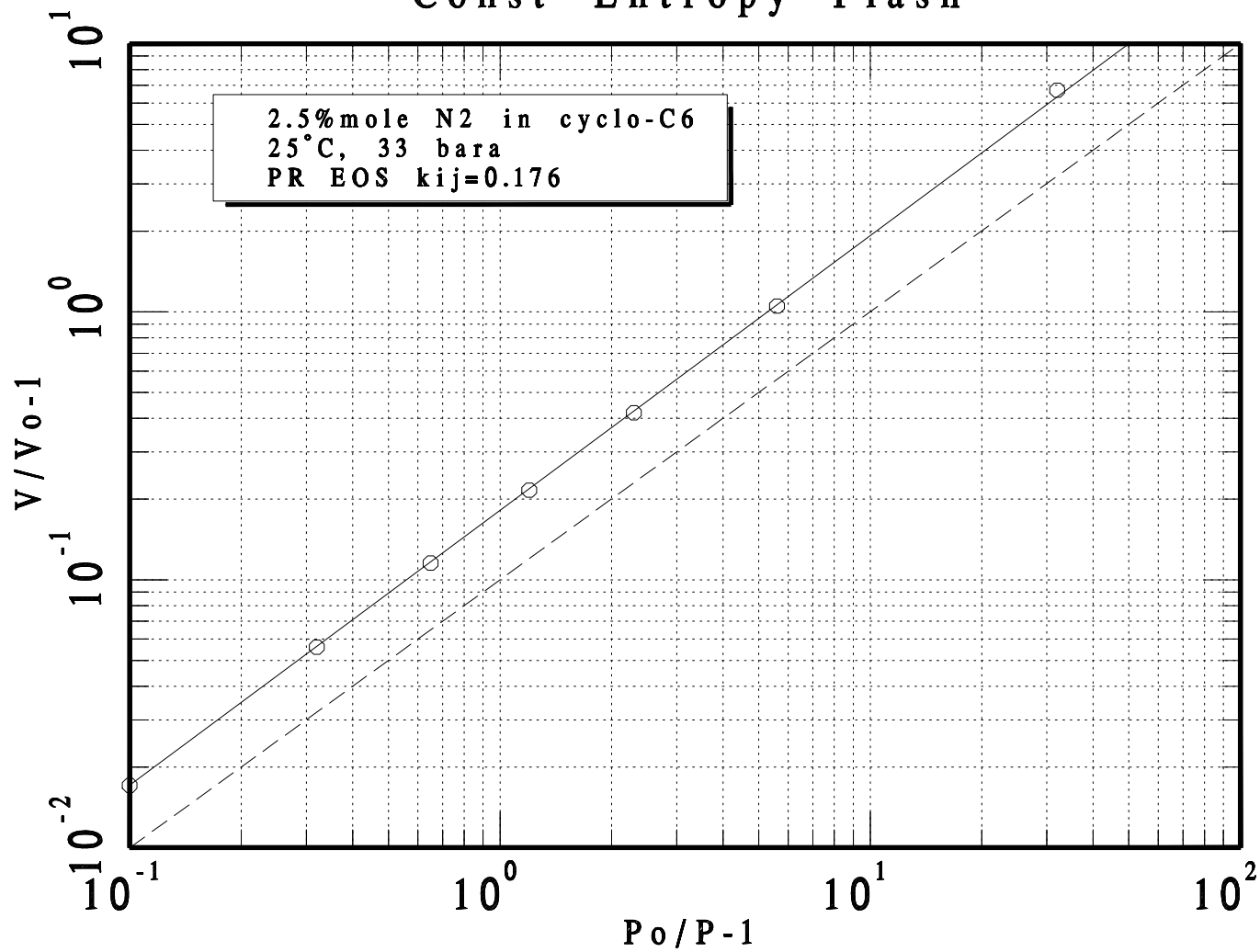
Best estimate $k_{12} = 0.176$ (PR EOS)

(caution - different value for SRK)

Case III – N₂ cyclo-C₆ const S flash

P	T	X (N ₂)	Y (N ₂)	VL	VV	L/F mole
bar	C	mole fr	mole fr	m ³ /kmol	m ³ /kmol	liq/feed
33	25.00	0.025	0.9936	0.10187	0.74148	0.99999
30	24.96	0.0228	0.9932	0.1020	0.81612	0.99773
25	24.87	0.0191	0.9923	0.10228	0.98041	0.99395
20	24.76	0.0154	0.9911	0.10254	1.22696	0.99013
15	24.62	0.0116	0.9889	0.10279	1.6380	0.98627
10	24.41	0.00774	0.9845	0.10304	2.4599	0.98233
5	24.02	0.00384	0.9714	0.10328	4.9233	0.97813
1	22.6	6.92E-4	0.8727	0.10335	24.5323	0.97212

Const Entropy Flash



Case III N₂ / Cyclo-C6 Preliminary Solution (Long Form)

PR EOS (2.5% mole N₂ in cyclo C6)

33 bara, 25°C, x_o = 0.0 (0.0001)

Constant S flash to lower pressures

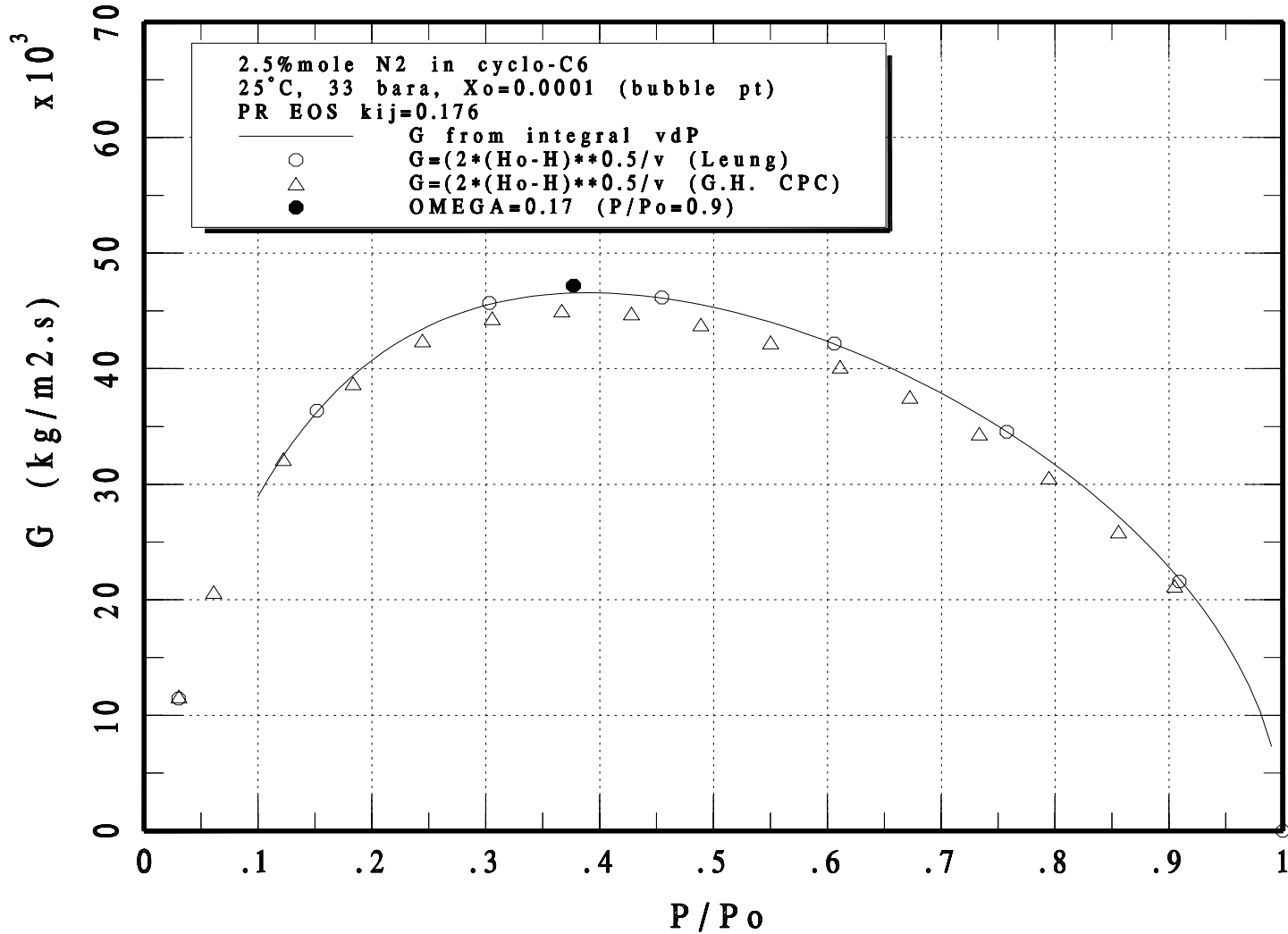
Temperature stays relatively unchanged (expected)

Curve fit P-v data

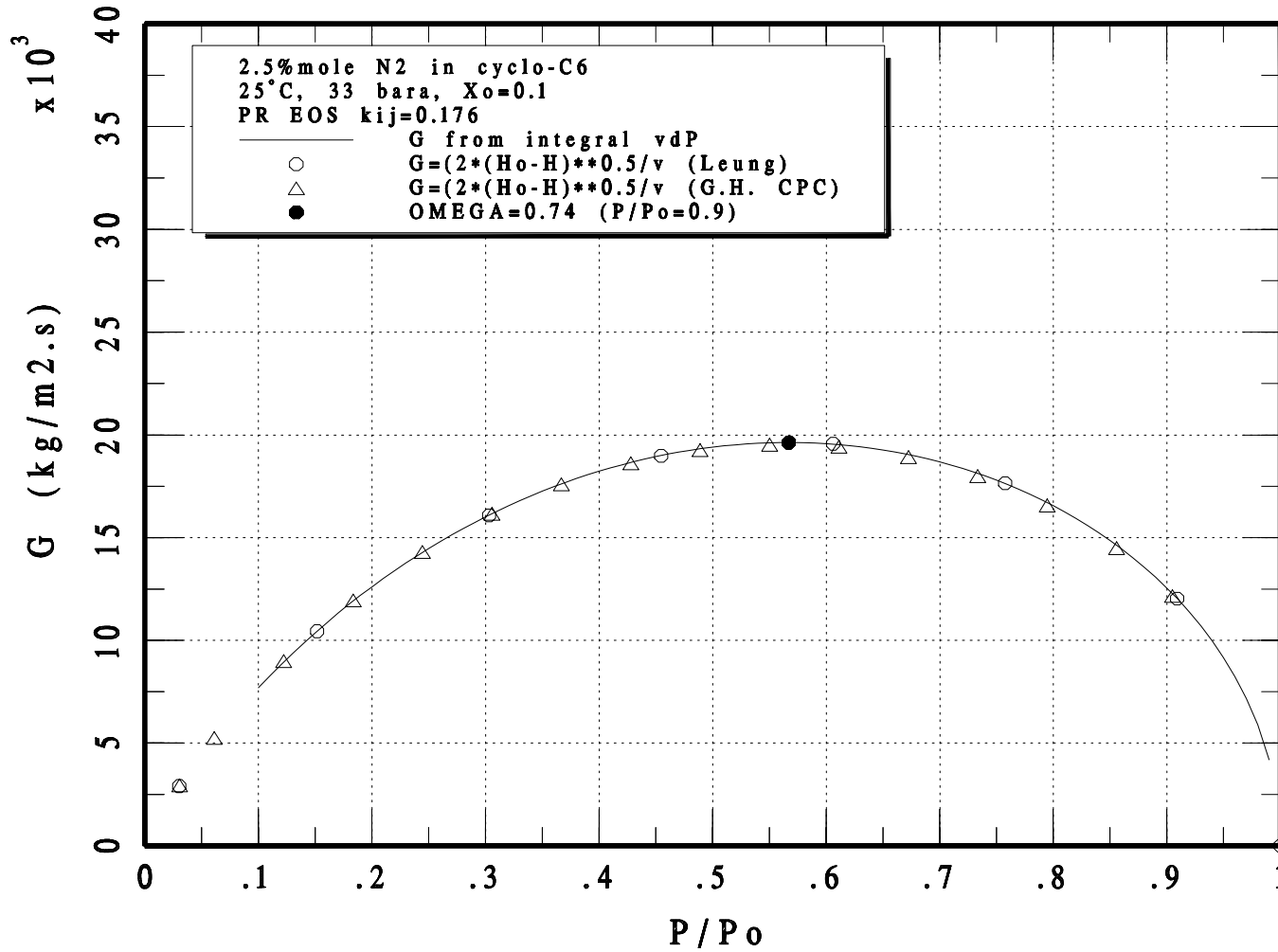
$$\left(\frac{v}{v_o} - 1 \right) = 0.181 \left(\frac{P_o}{P} - 1 \right)^{1.025}$$

Numerical integration of $\left[2 \int_{P_o}^{P_t} -v \, dP \right]^{1/2} / v_t$ to seek
max G

FLOW THRU NOZZLE (Const S)



FLOW THRU NOZZLE (Const S)



100% c-C6, 10 bar, 182.3°C (selected data)

Method (initials)	Case Ia $x_o = 0.0001$		Case Ib $x_o = 0.01$		Case Ic $x_o = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
ω_o (JL - stagnation)	6140	0.863	6020	0.849	5190	0.778
$\omega_{0.9}$ (JL - 0.9)	6160	0.863	6020	0.849	5210	0.777
$\omega_{0.9}$ (DS - 0.9)	6090	0.862	5970	0.850	5190	0.779
vdP (JL - numerical)	6120	0.870	5990	0.857	5180	0.780
vdP (DS - numerical)	6070	0.870	5990	0.857	5170	0.790
vdP (DS - Simpson)	6070	0.865	5940	0.854	5170	0.784
Aspen ΔH (GH)	5940	0.845	5810	0.835	5100	0.794
VENT (NP)	6150	0.867	6010	0.853	5200	0.795
SIMSCI ΔH (WG)	6050	0.860	NA	NA	NA	NA
SUPERCHEM vdP (EK)	6060	0.875	5930	0.850	5140	0.775
vdP (HF)	5850	0.873	5720	0.862	4940	0.793
Mean Value	6060	0.864	5940	0.852	5150	0.785
Std. Dev.	94	0.008	99	0.007	80	0.008

20% C2/n-C7 (Liq. Comp.), 10 bar, 51.9°C (selected data)

Method (initials)	Case IIa $x_0 = 0.0001$		Case IIb $x_0 = 0.01$		Case IIc $x_0 = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
$\omega_{0.9}$ (JL)	9850	0.762	8940	0.733	5650	0.652
$\omega_{0.9}$ (DS)	9810	0.748	8930	0.722	5660	0.648
vdP (JL – numerical)	9780	0.768	8890	0.738	5650	0.650
vdP (DS – numerical)	9770	0.748	8890	0.730	5650	0.658
vdP (DS - Simpson)	9770	0.753	8890	0.727	5650	0.650
CHEMCAD (DS)	8370	0.709	8890	0.723	5660	0.649
ASPEN ΔH (GH)	9700	0.783	8800	0.731	5600	0.659
VENT (NP)	9180	0.750	8500	0.717	5670	0.640
SIMSCI ΔH (WG)	9870	0.755	NA	NA	NA	NA
SUPERCHEM vdP (EK)	9960	0.750	8980	0.725	5690	0.651
vdP (HG)	9840	0.756	8880	0.730	5510	0.656
Mean Value	9630	0.753	8860	0.728	5640	0.651
Std. Dev.	463	0.018	135	0.006	51	0.006

2.5% N₂/c-C6 (Liq. Comp.), 33 bar, 25°C (selected data)

Method (initials)	Case IIIa $x_0 = 0.0001$		Case IIIb $x_0 = 0.01$		Case IIIc $x_0 = 0.1$	
	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c	G_c (kg/m ² s)	η_c
$\omega_{0.9}$ (JL)	47190	0.377	38050	0.456	19630	0.567
$\omega_{0.9}$ (DS)	46000	0.375	37440	0.452	20050	0.564
vdP (JL - numerical)	46600	0.390	37800	0.460	19630	0.566
vdP (DS - numerical)	45740	0.379	37330	0.457	20060	0.554
vdP (DS - Simpson)	45700	0.383	37320	0.455	20060	0.563
CHEMCAD(DS)	45750	0.382	37330	0.455	19960	0.566
ASPEN ΔH (GH)	45000	0.382	37000	0.458	19550	0.565
VENT (NP)	44800	0.355	40730	0.403	22040	0.494
SIMSCI ΔH (WG)	44990	0.385	NA	NA	NA	NA
SUPERCHEM (EK)	45850	0.379	37600	0.455	19570	0.561
Mean	45760	0.379	37840	0.450	20060	0.556
Std. Dev.	740	0.009	1120	0.018	770	0.023

Uncertainties in various models

- Case II bubble point calculation – 61.7°C (RD) vs 51.9°C(EK), perhaps due to different k_{ij} value, however G not much affected.
- NIST, DIPPR, PR-EOS would yield close agreement in P-T saturation, but not always in liquid density, however G not much affected.
- Chemcad uncertainty in its calculation method (DS will follow up).

FINDINGS

- Good agreement so far for all cases. Flash results are quite consistent.
- G based on ΔH from const S flash and integral vdP should yield consistent result, even for multicomponent system.
- Integral vdP method much more forgiving, even const H flash would yield close enough solution (not true approaching high quality flow).
- N₂-cHex case PR $k_{ij} = 0.176$ and NRTL model yield quite similar solutions.

Next DIERS UG Round-Robin

- Non-equilibrium (or frozen) nozzle flow ?
- Slip-equilibrium nozzle flow?
- Extend to pipe flow of various length ?
- Supercritical discharge?

Next Benchmark

- Pipe discharge mass flux calculations
- HEM (homogeneous-equilibrium model) assumption.
- Same 3 systems, 3 inlet qualities

$$x_o = 0.0001, 0.01, 0.1$$

- 2 pipe configurations -

$$N = K_{en} + 4f_F L/D = 1.5 \quad (L/D = 50)$$

$$N = K_{en} + 4f_F L/D = 5 \quad (L/D = 225)$$

fully turbulent two-phase, $f_F = 0.005$ (Fanning)

sharp-edge entrance, $K_{en} = 0.5$